

Inferring model parameters in markets with collars

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Abstract

Security prices are set by a continuous auction, the rules of which are set by the exchange or by the government. For many exchanges, there is a general free-flow of price information resulting in stock prices which can be modelled by a random walk following a Weiner-Levy process. However, many markets have collars, under which the rules of the auction will not let prices move too rapidly. In this paper we present methods for estimating the volatility of the underlying price data when the true price information is obscured by such collars. Numerical simulations are presented which demonstrate and contrast the methods.

Key words and phrases. Estimation of volatility, market models, market collars.

1 Introduction

Security prices are set by a continuous auction. The rules of the auction are set by the exchange or by the government. For many exchanges, there is a general free-flow of price information resulting in stock prices which can be modelled by a random walk following a Weiner-Levy process, perhaps with an added component of drift depending on the season, investor psychology, or other poorly understood factors. This model of stock prices has been very important both theoretically and practically to the development of option pricing.

However, many markets have collars, under which the rules of the auction will

not let prices move too rapidly. We consider the case where the previous day's closing price is set as the center of a percentage band outside of which the stock will not trade for that day. Since this affects the volatility of stock prices, and options are priced as a function of volatility, the question arises as to the proper pricing of options in markets with collars.

In this paper we present three methods by which the volatility of the ideal price can be estimated by looking only at the exchange price: a method based on the measure of waiting times; a method based on the estimate of the likelihood of an at-market day being followed by an at-limit day (definitions follow); the renewal time method introduced by Chiang and Wei. [?]. In the full paper we provide a rigorous proof of strong consistency for their method.

In addition, we compare the results of these methods to the volatility of the actual stock price and perform simulations on which of the two values are more appropriate for option pricing.

2 Methods

2.1 Definitions

Our ideal stock trace is a sequence of closing prices S_0, S_1, \dots . The inter-day increments are independent, identically distributed random variables with log-normal distribution of common mean μ and variance σ^2 ,

$$Z_i = \ln(S_i/S_{i-1}) \sim N(\mu, \sigma^2).$$

If r is the risk-free rate, then $\mu = r - \sigma^2/2$.

Collars are applied to the ideal price trace S_i to create an observable sequence S_i^* of exchange prices. Given lower and upper collars,

$$\kappa_l < 1 < \kappa_u,$$

we define S_i^* , $i = 0, 1, 2, \dots$ according to,

$$S_i^* = \begin{cases} S_0 & \text{if } i = 0, \\ \kappa_l S_{i-1}^* & \text{if } S_i < \kappa_l S_{i-1}^*, \\ \kappa_u S_{i-1}^* & \text{if } S_i > \kappa_u S_{i-1}^*, \\ S_i & \text{otherwise.} \end{cases}$$

We classify each observation S_i^* as either *at-market* or *at-limit*. Since we cannot always observe S_i , an observation S_i^* will be considered at-market only if it is