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Midterm

OCTOBER 9, 2006. 11:15–12:05 PM

There are five problems each worth five points for a total of 25 points. Show all your work, partial credit will be awarded. Space is provided on the test for your work; if you use a blue book for additional workspace, sign it and return it with the test. No notes, no collaboration.

Name: _____

Problem	Credit
1	
2	
3	
4	
5	
Total	

1. Order the following functions by Big-Oh,

(a) n^2 ,

(b) 1 (a constant),

(c) $10n \log n$,

(d) $2^n + \log n$,

(e) \sqrt{n} ,

(f) $n^3 - 40n$,

(g) $2n^2 + 7n - 4$,

(h) $20n + 1$.

2. Consider the following code,

```
sum = 0
for i = 1 to length(A)
    sum = sum + A[i]
end for
```

Write a loop invariant which shows that this code correctly calculates the sum of the values stored in $A[1]$, $A[2]$, \dots , $A[n]$. Prove correctness using the invariant.

3. Show the result of **Build-Max-Heap** when run on the following array. (Show several intermediate steps.)

13	16	14	15	1	8	7	3	9	10
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4. Consider the following algorithm to see if all the values in an array are unique,

```
for i = 1 to length(A)
  for j = 1 to length(A)
    if ( i!=j )
      if A[i] == A[j] return false
    end for
  end for
return true
```

Give the run time of this algorithm, in terms of $n = \text{length}(A)$. Justify your answer.

Find another algorithm to accomplish the task of finding if all values in an array are unique, which is significantly faster than the algorithm presented above.

5. When you are one year old, one candle is lit for your birthday. When you are two years old, two candles are lit. When three, three candles, and so forth. By your 80-th birthday, what is the total number of candles lit for all your birthdays combined, including the 80 candles for this birthday.