Lecture 8 21 September 1993

1. Went over solution set 3.
2. Stacks: example from Sedgewick's book on arithmetic evaluation. Using linked list as stack.
3. **Trees.**
4. Terminology: vertices or nodes, edges are pairs of nodes.
5. Path: sequence of distinct nodes such that successive nodes in list are edges. Also, single node is a path.
6. Root: a distinguished node.
7. Tree: for every vertex there is a unique path from the root to the vertex.
8. Direction: from root to vertex. Parent and children.
9. Subtree: vertex (as new root) and all its descendants.
10. Ordered tree, forest, full etc.
11. Properties: 1-5 from Sedgewick. However, the first was more difficult to establihs than I thought. Here is how.

    Theorem: In a tree there is a unique path between any two nodes:

    Proof: Existence. Let r be the root, and v_1, v_2 the vertices. If either or both $v_i = r$ done.

    So let p_i be paths from v_i to r and choose π to be the vertex which is the last time the p_i intersect as one travels from r to v_2. Then $v_1 - \pi - v_2$ is a path.

    Uniqueness: Suppose π_i for $i = 1, 2$ are two distinct paths from v to w. Follow both paths from v until it splits. Replace v with the node at which it splits. Hence (v, v_i) begins π_i and $v_1 \neq v_2$. Let w^* be the first time π_2 hits π_1 (as one moves from v to w). Replace w by w^*. Hence π_i are a simple cycle (at least three edges) joining w and v.

    If the root is on the cycle, then there are two paths from the root to any other node on the cycle by traversing the cycle clockwise or counter-clockwise..

    If the root is not on the cycle, choose any node of the cycle and connect it to the root. Then working back, let r^* be the first node on the path from the root to the cycle which hits the cycle. Replace the route with r to r^*. There are two distinct paths from the root to anything on the cycle, consisting of the path from r to r^* then traversing the cycle clockwise or counter-clockwise.