Burton Rosenberg

Test 1

FEBRURARY 24, 5:00–6:15

There are four problems each counting equally.

Name: ______________________________

Problem	Credit
1	
2	
3	
4	
Total	

On my honor, I have neither given nor received aid on this examination-assignment.

Signature: ______________________________

1. [SIMPLEX METHOD]

   Solve the Following LP showing step-by-step the simplex method:

$$
\begin{array}{llllllll}
\max & x_1 & + & 2x_2 & + & x_3 & & \\
\\
\text{s.t.} & x_1 & + & x_2 & + & x_3 & \leq & 2 \\
 & x_1 & + & x_2 & & & \leq & 1 \\
\\
 & x_1, & x_2, & x_3 & \geq & 0 & &
\end{array}
$$

2. [DUALITY]

   (a) Give the Dual of the previous LP problem.

   (b) Find the optimal dual solution, using whatever method you wish.

   (c) Demonstrate the Complementary Slackness conditions for your optimal dual/primal solution pair. That is, what should be true and what is true for each of the 5 variable-inequality pairings.

3. [LU DECOMPOSITION]

(a) Use Gaussian Elimination with partial pivoting to decompose,

$$A = \begin{bmatrix} 1 & 1 & 1 \\ 2 & 0 & 2 \\ 0 & 3 & 3 \end{bmatrix},$$

into the product,

$$L_3 P_3 L_2 P_2 L_1 P_1 A = U,$$

where L_i are column i eta-matrices, P_i are permutation matrices, and U is upper triangular with 1's down the diagonal.

(b) Use back substitution and your decomposition to find x_1, x_2, x_3 real numbers which satisfy,

$$\begin{array}{ccccccc} x_1 & + & x_2 & + & x_3 & = & 1 \\ x_1 & & & + & x_3 & = & 1/2 \\ & & x_2 & + & x_3 & = & 1/3 \end{array}$$

4. [Theory]

   Prove that the product AB of two square matrices is nonsingular if and only if both A and B are nonsingular.