Burton Rosenberg

Test 2

April 28, 5:00–6:15

There are five problems each counting equally.

Name: ____________________

Problem	Credit
1	
2	
3	
4	
5	
Total	

On my honor, I have neither given nor received aid on this examination-assignment.

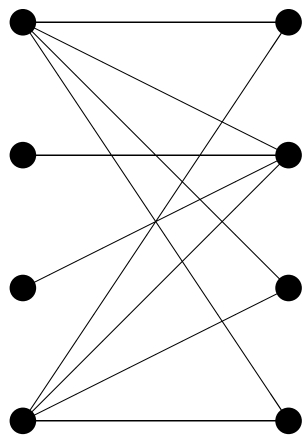
Signature: ____________________

1. [NETWORK FLOWS] Negative Cycles

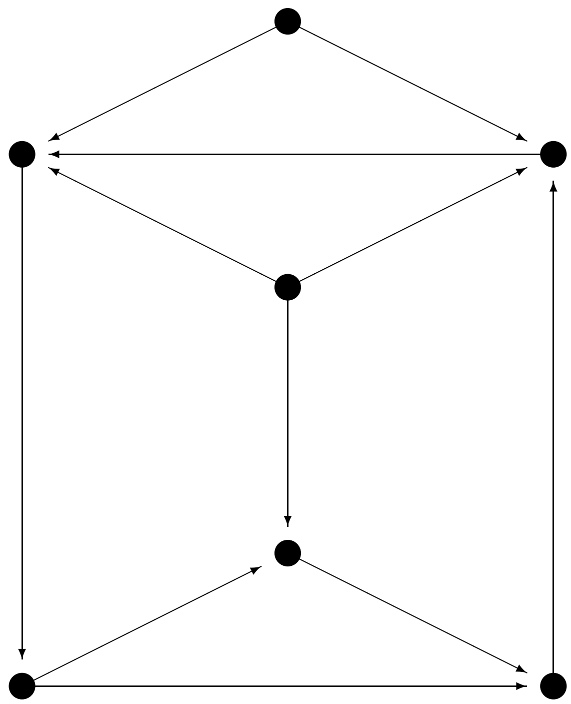
   You are given a network $G(V, E)$ with edge costs $w(e)$ for any $e \in E$. These costs can be negative or positive. You would like to find out if there exists a cycle of edges in G such that the weight summed over the edges in the cycle is negative.

   Express this as a condition on a network transport problem.

2. [MAXIMUM MATCHING] Find a maximum matching and minimum cover in the following graph:



3. [NODE COSTS AND TREE FEASIBLE SOLUTIONS] Give a spanning tree and assign costs, $\{\, y_v \mid v \in V \,\}$ to the following network. *Assume all edge costs are 1.*



4. [APPLICATIONS] Setup the following problems as a network transport problem.

   You are given a network of computers, that is, a graph $G = (V, E)$ where V, the set of nodes, represent computers, and E, the set of edges, giving communication pathways between computers. Each edge $e \in E$ has a weight, $w(e)$, giving the costs of communication across the pathway. Some of the computers are servers, which we represent as a subset S of V. Each non-server, $v \in V \backslash S$ has a demand for disk blocks $b(v)$. A non-server computer $v \in V \backslash S$ will get its blocks over the network from a server by selecting a path to a nearby server, call it $P(v)$, and paying $w(v)b(v)$, where $w(v)$ is the sum of the weights of all edges in the path $P(v)$:

   $$w(v) = \sum_{e \in P(v)} w(e).$$

   The total disk blocks demanded is:

   $$D = \sum_{v \in V \backslash S} b(v).$$

   You are to allocate the D blocks to the servers in S such that the total communication costs are lowest, summed over all non-server computers:

   $$\sum_{v \in V \backslash S} w(v).$$

5. [THEORY] Prove that a connected graph on n vertices is a tree if and only if it has $n - 1$ edges.