MATH 688: THEORY OF COMPUTABILITY AND COMPLEXITY _____1

Resume

DATE: 13 October, 1992

- 1. While-programs.
 - (a) Syntax.
 - (b) Macros, appropriate and inappropriate use of macros.
 - (c) Variable relabeling.
 - (d) Equivalence with other "program forms".
- 2. Relationship between while-programs and functions.
 - (a) Partial functions, domain of definition and extension ordering.
 - (b) Input/output conventions for arity k functions.
 - (c) Non-halting of a program versus undefinedness of a function.
 - (d) Church's thesis, the definition of a computable function.
 - (e) Are all functions computable?
- 3. Enumeration of computable functions.
 - (a) Definition of countability: in bijection with the naturals.
 - (b) Counting while-programs, encodings.
 - (c) Introduction to uncountability.
 - i. The uncountability of the reals and of partial functions.
 - ii. Cantor diagonalization.
 - (d) Cardinality argument for the existence of uncomputable functions.
 - (e) The halting function is uncomputable.
- 4. Universal Functions.
 - (a) Program rewriting: the string operations head, tail and concatenation as number-theoretic functions.
 - (b) Syntax checking.
 - (c) Simulation of unbounded memory.

- i. Pairing functions: a bijection from pairs of naturals to the naturals.
- ii. Extensions of pairing functions to n-tuples and ∞ -tuples of finite support.
- (d) The universal function is computable.

- Second Half of the Course.

- 5. Recursion
 - (a) Recursive programs are while-program computable.
 - i. Posets of functions by extension ordering and least-upperbounds.
 - ii. Recursive programs are least-upper-bounds.
 - iii. Effective least-upper-bounds are computable.
 - (b) The Recursion Theorem, Section 6.1.
- 6. Acceptable Programming Systems.
 - (a) Roger's Isomorphism Theorem, Section 6.2.
 - (b) Turing Machines.
 - (c) Recursive Functions.
 - i. Primitive recursive functions and loop-programs.
 - ii. Unbounded minimization and partial recursive functions.
 - (d) Undecidability of certain word problems.
- 7. The Theory of NP-Completeness
 - (a) Deterministic and Non-deterministic Turing machines.
 - (b) Polynomial-time transformations. Classes P and NP.
 - (c) SAT is NP-Complete.
 - (d) Other NP-Complete problems.