Unrolling the recursion in getLinear1

Recall the recursion of the filter in getLinear1

$$y[t+1] = y[t] + \frac{1}{\tau}(s[t] - y[t]), \tag{1}$$

where $y[t]$ is the linear output of the neuron model, and $s[t]$ the input stimuli. Rearranging the *r.h.s.* terms yields:

$$y[t+1] = \frac{1}{\tau}s[t] + \left(1 - \frac{1}{\tau}\right) y[t]. \tag{2}$$

Also notice that:

$$y[t] = \frac{1}{\tau}s[t-1] + \left(1 - \frac{1}{\tau}\right) y[t-1]. \tag{3}$$

Replacing (3) in (2) yields:

$$y[t+1] = \frac{1}{\tau}s[t] + \left(1 - \frac{1}{\tau}\right) \frac{1}{\tau}s[t-1] + \left(1 - \frac{1}{\tau}\right)^2 y[t-1]. \tag{4}$$

Let's repeat this replacement one more time:

$$y[t+1] = \frac{1}{\tau}s[t] + \left(1 - \frac{1}{\tau}\right) \frac{1}{\tau}s[t-1] + \left(1 - \frac{1}{\tau}\right)^2 \frac{1}{\tau}s[t-2] + \left(1 - \frac{1}{\tau}\right)^3 y[t-2]. \tag{5}$$

This can be done N times:

$$\begin{aligned} y[t+1] \;=\; & \frac{1}{\tau}s[t] + \left(1 - \frac{1}{\tau}\right) \frac{1}{\tau}s[t-1] + \left(1 - \frac{1}{\tau}\right)^2 \frac{1}{\tau}s[t-2] + \cdots \\ & + \left(1 - \frac{1}{\tau}\right)^N \frac{1}{\tau}s[t-N] + \left(1 - \frac{1}{\tau}\right)^{N+1} y[t-N]. \end{aligned} \tag{6}$$

Performing this unrolling process indefinitely yields:

$$y[t+1] = \frac{1}{\tau}\sum_{n=0}^{\infty} \left(1 - \frac{1}{\tau}\right)^n s[t-n]. \tag{7}$$

Taking $w[n] = \left(1 - \frac{1}{\tau}\right)^n$, equation (8) becomes

$$y[t+1] = \frac{1}{\tau}\sum_{n=0}^{\infty} w[n]s[t-n], \tag{8}$$

which corresponds to the convolution operation we discussed in the lab.