Final

Friday, 7 May 2021
11:00 AM –1:30 PM

- There are 7 problems each worth 6 points and 1 extra credit problem worth 3 points.
- As an important protocol for today's test, please turn your cameras on.
- While situations differ, every student is responsible for ensuring the integrity of the test, and must take all reasonable steps in support of ensuring the integrity.
- No notes, no collaboration. Please help the evaluation of partial credit by showing your work towards the solution. Do not be overly concerned with the challenge problems. Do your own work.
- Please sign the cover page so show agreement with these directions.

Name: ______________________________

Problem	Credit
1	
2	
3	
4	
5	
5–ec	
6	
7	
Total	

1. Reduce the following SAT instance to a 3SAT instance.

$$\neg\Big(\ (a \wedge b \wedge c \wedge d)\ \ \vee \neg(a \vee (b \wedge c))\ \Big)$$

2. Give a Turing Machine that accepts the language of strings over $\{\$, 0, 1\}$ of strings that begin with a $ and are followed by an equal number 0's and 1's.

   You may assume that the only characters in the string before the first blank is the leading $ then only 0's and 1's.

3. Put a (✓) the box that *most precisely* describes the language.

	Rec.	**R.E.**	**non–RE**
A_{DFA}			
A_{REX}			
coA_{CFG}			
EQ_{DFA}			
A_{TM}			
EQ_{TM}			
$coEQ_{TM}$			
$coHALT_{TM}$			
coA_{TM}			
$REGULAR_{TM}$			

The languages are,

$$\begin{aligned}
A_{CFG} &= \{\, \langle G, w\rangle \mid G \text{ is a CFG that generates string } w \,\} \\
A_{DFA} &= \{\, \langle B, w\rangle \mid B \text{ is a DFA that accepts input string } w \,\} \\
A_{REX} &= \{\, \langle R, w\rangle \mid R \text{ is a regular expression that generates string } w \,\} \\
A_{TM} &= \{\, \langle M, w\rangle \mid M \text{ is a TM and } M \text{ accepts } w \,\} \\
EQ_{DFA} &= \{\, \langle A, B\rangle \mid A \text{ and } B \text{ are DFAs and } L(A) = L(B) \,\} \\
EQ_{TM} &= \{\, \langle M_1, M_2\rangle \mid M_1, M_2 \text{ are TMs and } L(M_1) = L(M_2) \,\} \\
HALT_{TM} &= \{\, \langle M, w\rangle \mid M \text{ is a TM and } M \text{ halts on input } w \,\} \\
REGULAR_{TM} &= \{\, \langle M\rangle \mid M \text{ is a TM and } L(M) \text{ is a regular language } \}
\end{aligned}$$

For a language X the language coX is the language of the complement set of X.

4. The k–clique problem is given a graph with vertex set V of size n and edge set E of size m, is there a subset K of the vertex set V, such that K is of size k and K is a clique, that is, every pair of vertices in K are connected by an edge.

   (a) Give an algorithm to solve the problem. Analyze the the algorithm runtime.

   (b) It is likely your algorithm is not polynomial time. Is there a polynomial time algorithm for this problem?

5. (a) Show that P is closed under union, concatenation and complement.

   (b) *Extra Credit Problem:* Show that P is closed by star.

6. Consider an polynomial in one variable with integer coefficients,

$$p(x) = a_d x^d + a_{d-1} x^{d-1} + \ldots + a_0$$

The notation for any such polynomial is $p \in Z[x]$. The language R is the set of such polynomials that have an integer root,

$$R = \{\, p \in Z[x] \mid \text{ there is a } r \in Z \text{ such that } p(r) = 0 \,\}$$

(a) Give a P-time verifier for the set R.

(b) Give a decider for the set R. (Do not be concerned with runtime, except it must always decide in finite time.)

7. Show that,

   $$CFL_{TM} = \{ \langle M \rangle \mid M \text{ is a TM and } L(M) \text{ is a context free language } \}$$

   is undecidable.