

February 26, 2025

Zoom lecture.

Chomsky Normal Form

① $S \rightarrow \epsilon$

② $A \rightarrow a$

③ $C \rightarrow AB$

④ no S on RHS

$$S \rightarrow aSb \mid \epsilon$$

ϵ -removal (① \rightarrow ④)

$$S \rightarrow \epsilon \mid aTb \mid ab$$

$$T \rightarrow ab \mid aTb$$

Formalities (② \rightarrow ③)

$$S \rightarrow a \mid AB \mid ATB$$

$$T \rightarrow AB \mid ATB$$

$$A \rightarrow a$$

$$B \rightarrow b$$

$$S \rightarrow \epsilon \mid AB \mid ATB$$

$$T \rightarrow AB \mid ATB$$

$$A \rightarrow a$$

$$B \rightarrow b$$

==

$$S \rightarrow \epsilon \mid AB \mid AU$$

$$T \rightarrow AB \mid AU$$

$$U \rightarrow TB$$

$$A \rightarrow a$$

$$B \rightarrow b$$

Reg Expr

$(b | a b^* a)^*$

$S \rightarrow \epsilon | TS$

$T \rightarrow b | aBa$

$B \rightarrow \epsilon | LB$

most
natural

in my
opinion

$$S \rightarrow \epsilon \mid TS$$

$$T \rightarrow b \mid aBa$$

$$B \rightarrow \epsilon \mid LB$$

ϵ -reduced (1) q (4)

$$S \rightarrow \epsilon \mid TR \mid T$$

$$R \rightarrow T \mid TR$$

$$T \rightarrow b \mid a \mid aBa$$

$$B \rightarrow b \mid LB$$

$$S \rightarrow \epsilon \mid TR \mid T$$

$$R \rightarrow T \mid TR$$

$$T \rightarrow b \mid a \mid a^B a$$

$$B \rightarrow b \mid bB$$

$$S \rightarrow \epsilon \mid TR \mid b \mid AA \mid ABA$$

$$R \rightarrow TR \mid b \mid AA \mid ABA$$

$$T \rightarrow b \mid AA \mid ABA$$

$$B \rightarrow b \mid CB$$

$$C \rightarrow b$$

$$A \rightarrow a$$

$S \rightarrow \varepsilon \mid TR \mid b \mid AA \mid ABA.$

$R \rightarrow TR \mid b \mid AA \mid ABA$

$T \rightarrow b \mid AA \mid ABA$

$b \rightarrow b \mid CB$

$c \rightarrow b \quad A \rightarrow a$

$S \rightarrow \varepsilon \mid b \mid AA \mid AU \mid TR$

$R \rightarrow b \mid AA \mid AU \mid TR$

$T \rightarrow b \mid AA \mid AU$

$U \rightarrow BA$

$b \rightarrow b \mid CB$

$c \rightarrow b \quad A \rightarrow a$

$S \rightarrow \epsilon \mid b \mid AA \mid AU \mid TR$

$R \rightarrow b \mid AA \mid AU \mid TR$

$T \rightarrow b \mid AA \mid AU$

$U \rightarrow BA$

$b \rightarrow b \mid c \mid B$

$c \rightarrow b \quad A \rightarrow c$

all 2-letter strings

$S \rightarrow AA \rightarrow c^2$

$\rightarrow TR \rightarrow bb$

$$S \rightarrow \epsilon \mid b \mid AA \mid AU \mid TR$$

$$R \rightarrow b \mid AA \mid AU \mid TR$$

$$T \rightarrow b \mid AA \mid AU$$

$$U \rightarrow BA$$

$$b \rightarrow b \mid c \mid B$$

$$c \rightarrow b \quad A \rightarrow c$$

all 3-letter strings

$$S \rightarrow AU \rightarrow aBA \rightarrow aba$$

$$S \rightarrow TR \rightarrow bAA \rightarrow ba a$$

$$S \rightarrow TR \rightarrow AA b \rightarrow aab$$

len 1 len 2 len 3

CYK - dynamic programming
algorithm

Itiroo Sakai 1961

Tadao Kasami 1965

Daniel Younger (1967)

John Cocke (1970)

$O(n^3 |G|)$ time decision

"organized brute search"

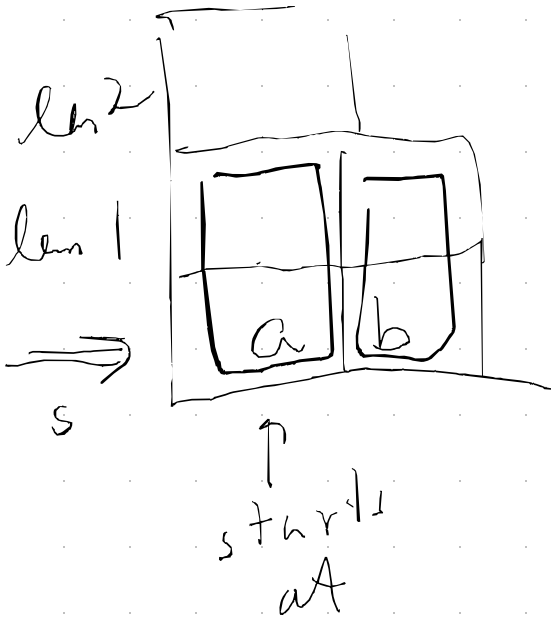
TABLEAU Richard Bellman

$S \rightarrow \epsilon \mid AB \mid AU$

$T \rightarrow AB \mid AU$

$W \rightarrow TB$

$A \rightarrow a \quad B \rightarrow b$



$S \rightarrow \epsilon \mid AB \mid AU$

$T \rightarrow AB \mid AU$

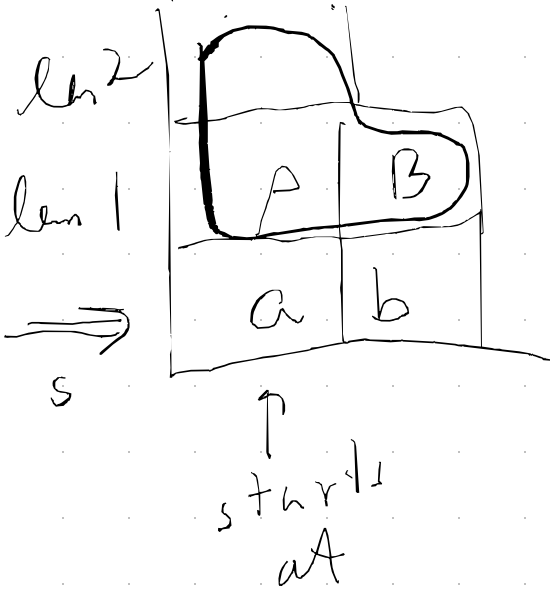
$W \rightarrow TB$

$A \rightarrow a \quad B \rightarrow b$

$? \rightarrow AB$

$? \in \{S, T\}$

should have T in this box too



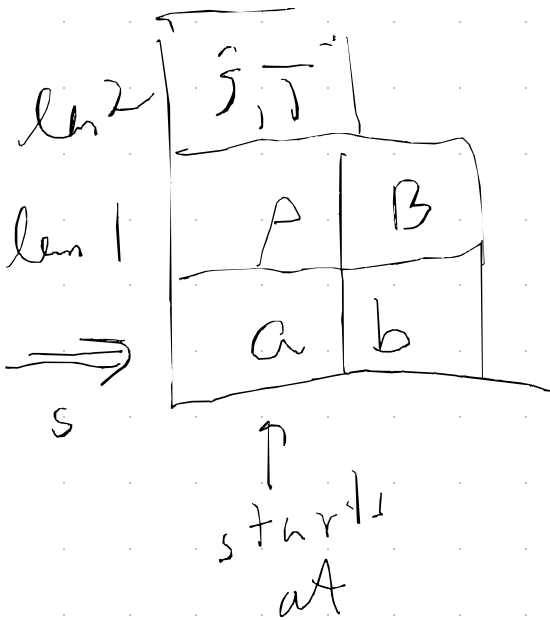
Tf

$S \rightarrow \epsilon \mid AB \mid AU$

$T \rightarrow AB \mid AU$

$W \rightarrow TB$

$A \rightarrow a \quad B \rightarrow b$

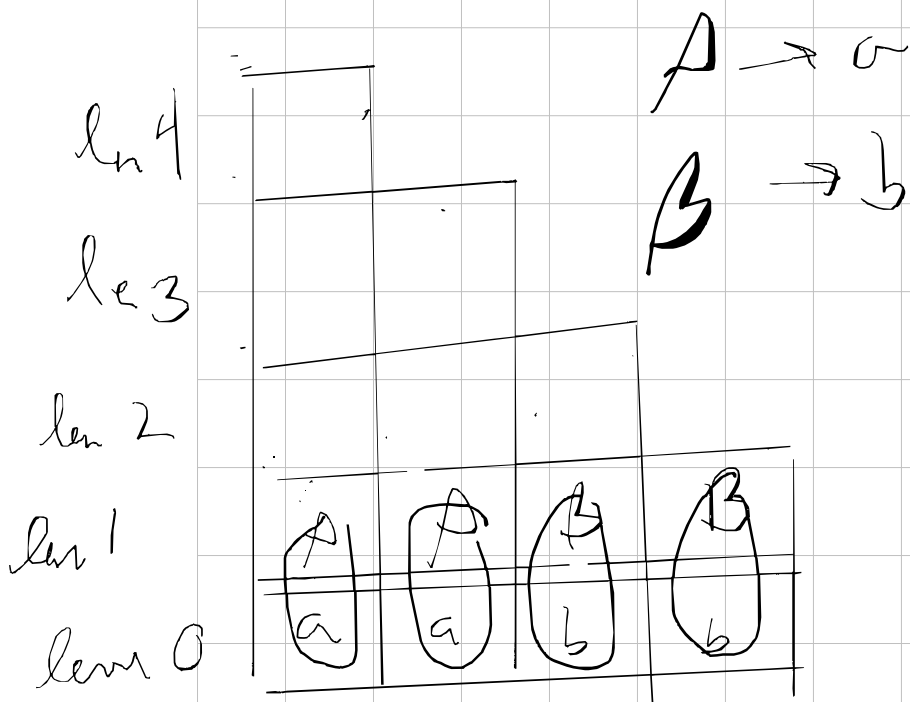


$S \rightarrow \epsilon \mid AB \mid AU$

$T \rightarrow AB \mid AU$

$U \rightarrow TB$

$A \rightarrow a \quad B \rightarrow b$



$S \rightarrow \epsilon \mid AB \mid AU$

$T \rightarrow AB \mid AU$

$U \rightarrow TB$

$A \rightarrow a \quad B \rightarrow b$

$? \rightarrow PA$

$? \rightarrow PB$

$? \rightarrow BB$

len 4

len 3

len 2

len 1

len 0

		S, T	\emptyset
\emptyset	A	B	B
a	a	b	b

$S \rightarrow \epsilon \mid AB \mid \Lambda U$

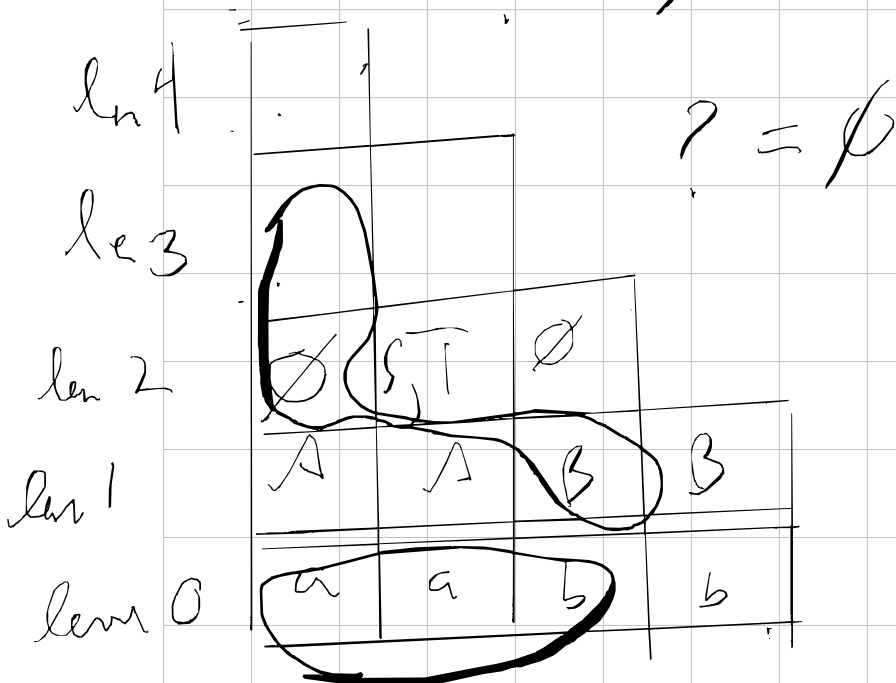
$T \rightarrow AB \mid \Lambda U$

$U \rightarrow TB$

$\Lambda \rightarrow a \quad B \rightarrow b$

$aaab \rightarrow aa|b$

$\uparrow \rightarrow \emptyset B$



$S \rightarrow \epsilon \mid AB \mid AU$

$T \rightarrow AB \mid AU$

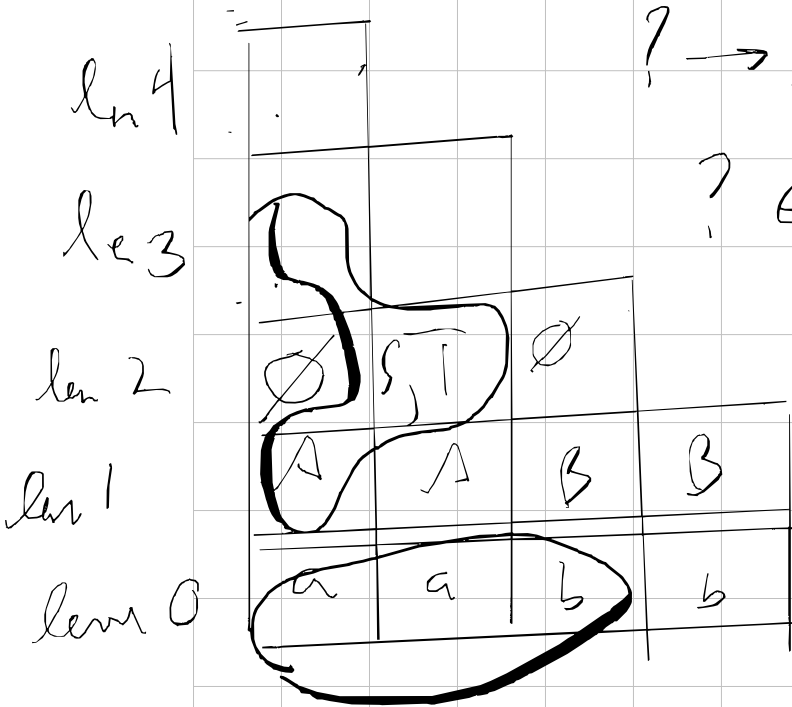
$U \rightarrow TB$

$A \rightarrow a \quad B \rightarrow b$

$aub \Rightarrow a \mid ub$

? $\rightarrow AS, AT$

? $\in \emptyset$



$S \rightarrow \epsilon \mid AB \mid AU$

$T \rightarrow AB \mid AU$

$U \rightarrow TB$

$A \rightarrow a \quad B \rightarrow b$

$abbb \Rightarrow A\phi$

$abbb \Rightarrow SB \cup TB$

$len 4 \Rightarrow \phi \cup U$

$len 3$

$len 2$

$len 1$

$len 0$

	\emptyset	U	
	\emptyset	S, T	\emptyset
	A	A	B, B
	a	a, b	b

$S \rightarrow \epsilon \mid AB \mid \Lambda U$

$T \rightarrow AB \mid \Lambda U$

$U \rightarrow TB$

$\Lambda \rightarrow a \quad B \rightarrow b$

$aab|b \rightarrow \emptyset \quad B \rightarrow \emptyset$

$aa|bb \rightarrow \emptyset \quad \emptyset \Rightarrow \emptyset$

$a|abb \rightarrow \Lambda U \Rightarrow T, S$

len 4

len 3

len 2

len 1

len 0

	S, T			
	• \emptyset	•• U		
	\emptyset	S, T	\emptyset	
	A	A	B	B
	a	a	b	b

$S \rightarrow \epsilon \mid AB \mid \Delta U$

$T \rightarrow AB \mid \Delta U$

$U \rightarrow TB$

$\Delta \rightarrow a \quad B \rightarrow b$

$\emptyset, A, \overline{AT}$

3	\emptyset		
2	\emptyset	S, T	
1	A	A	B
0	Q	a	b